MTH241 Fall 2024: Quiz 03

Name: Cowboy Bebop
UID:

Closed book, no calculator, show your work clearly.

1. (5pt) Let $\vec{F}(t) = e^t\vec{i} + \ln t\vec{j} + \frac{1}{t^2}\vec{k}$ and $g(t) = \sqrt{t}$. Find the domain and the derivative of $\vec{F} \circ g$. (Grading: **2pt**: working; **1pt**: $\vec{F} \circ g$; **1pt**: domain; **1pt**: derivative)

$$F\circ g(t) = \begin{bmatrix} e^{\sqrt{t}} \\ \ln(\sqrt{t}) \\ \frac{1}{|t|} \end{bmatrix}$$

Domain

$e^{\sqrt{t}} \rightsquigarrow t \geq 0$

$\ln(\sqrt{t}) \rightsquigarrow t > 0$

$\frac{1}{|t|} \rightsquigarrow t \neq 0$

$\Rightarrow$ Domain of $F\circ g = (0,\infty)$

Pedantic note: Since $t>0 \Rightarrow |t| = t \Rightarrow F\circ g(t) = \begin{bmatrix} e^{\sqrt{t}} \\ \ln(\sqrt{t}) \\ \frac{1}{t} \end{bmatrix} \rightsquigarrow (F\circ g)'(t) = \begin{bmatrix} \frac{1}{2\sqrt{t}} e^{\sqrt{t}} \\ \frac{1}{2t} \\ -\frac{1}{t^2} \end{bmatrix}$

2. (5pt) Let $\vec{F}(t) = 2t^3\vec{i} + e^t\vec{j} + \sin(t)\vec{k}$. Find $\int_0^\pi \vec{F}(t)dt$ (Grading: **2pt**: working; **3pt**: correct integral for each component)

$$F = \begin{bmatrix} 2t^3 \\ e^t \\ \sin(t) \end{bmatrix} \rightsquigarrow \int_0^\pi F(t)\,dt = \begin{bmatrix} \frac{t^4}{2}\Big|_0^\pi \\ e^t\Big|_0^\pi \\ -\cos(t)\Big|_0^\pi \end{bmatrix} = \begin{bmatrix} \frac{\pi^4}{2} \\ e^\pi - 1 \\ 0 \end{bmatrix}$$

Second page: